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DETERMINATION OF RESIDUAL STRESS DISTRIBUTIONS IN AUTOFRETTAGED TUBING: A DISCUSSION

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There is a long-standing interest in developing a capability to predict the distribution of residual stresses in thick-wall tubes after internal pressurization--autofrettage. Since autofrettage involves partial or full plastic deformation of the tube, any computation of stress under pressure and, hence of the post-pressurization residual stresses, depends upon the assumed yield criterion. The latter may or may not include the material's strain-hardening and/or strain-softening capabilities. The most commonly used

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criteria are those named after Tresca and Mises.

In the absence of exact solutions for plastic deformations, simplifying assumptions concerning the material's behavior are being made for the development of workable solutions, sometimes with the knowledge that certain physical principles are being violated.

Many suggested solutions to the problem of autofrettage assume that Tresca's yield criterion prevails. Recent attempts to treat a "modified Tresca's yield criterion" as Mises' yield criterion and/or attempts to add strain-hardening and/or strain-softening (Bauschinger effect) to Tresca's yield criterion are being questioned here.

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LIST OF SYMBOLS

a ≡ tube's bore radius

b ≡ tube's outer radius

c ≡ radius of an elastic-plastic interface upon pressurization

E ≡ material's modulus of elasticity

p ≡ pressure

p_i ≡ internal pressure at the tube's bore

p_o ≡ external pressure at the tube's outer diameter

r ≡ radial distance

u ≡ displacement

x ≡ coordinate's direction in a cartesian coordinate system

y ≡ coordinate's direction in a cartesian coordinate system

z ≡ coordinate's direction in either a cartesian or in a cylindrical coordinate system

δ ≡ differential

δ ≡ $1 - \nu + \nu^2$

ϵ ≡ strain

η ≡ $(1-2\nu)^2$

ν ≡ material's Poisson's factor

σ ≡ stress

σ_Y ≡ material's yield strength

σ_U ≡ material's ultimate strength

ϕ ≡ a stress function

ρ ≡ radius of elastic-plastic interface

<u>Subscripts</u>:

i ≡ at the tube's inner diameter

i ≡ a coordinate's plane and/or a coordinate's direction

j ≡ a coordinate's plane and/or a coordinate's direction

k ≡ a coordinate's plane and/or a coordinate's direction

o ≡ at the tube's outer diameter

r ≡ a coordinate's plane and/or a coordinate's direction in a cylindrical coordinate system

θ ≡ a coordinate's plane and/or a coordinate's direction in a cylindrical coordinate system

x ≡ coordinate's direction in a cartesian coordinate system

y ≡ coordinate's direction in a cartesian coordinate system

z ≡ a coordinate's plane and/or a coordinate's direction in either a cylindrical or a cartesian coordinate system

() ≡ a subscript inside parentheses indicates a specific geometrical location, i.e., $\sigma_{rr(a)} \equiv \sigma_{rr}$ @ $r = a$ or $\sigma_{\theta\theta(c)} = \sigma_{\theta\theta}$ @ $r = c$.

INTRODUCTION

In their paper, "Determination of Residual Stress Distribution in Autofrettaged Tubing," Stacey and Webster (ref 1) rightfully suggest that calculation procedures usually involve simplifying assumptions about material behavior which may limit their accuracies. On one hand, limiting their accuracies implies that the results may be acceptable for one purpose, but not for another. On the other hand, it also implies that they may be close enough in one range of variables, but not in another. For these reasons, established facts and/or established mathematical formulas should be clearly presented as such and distinguished from assumptions. Preferably, the presentation of the latter should be accompanied by their limitations. This author found some ambiguities and confusion in key points, rather than the needed clarifications in the subject paper (ref 1). Moreover, it is being suggested here that failure to identify the origin and the derivations of some basic equations led to an apparent misuse of some equations (presented in the subject paper (ref 1)). Furthermore, it should be pointed out that while the authors took pains to double- and triple-check their data, these were all confined to one size tube made of only one material and autofrettaged to only one level. (They compared residual stress measurements obtained through the use of several different methods, but applied them all to only one tube.) One should consider this when attempting to extrapolate the use of the suggested equations outside this narrow range.

[1] A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

ELASTIC VERSUS PLASTIC REGIONS

It is generally accepted that in the elastic region the stress-strain relation of an isotropic material follows the extended Hooke's law, Eq. (1), (Eqs. (18) and (19) (ref 2)) at every point in the elastic range.

$$\epsilon_{ij} = 2\,\frac{1+\nu}{E}\,\sigma_{ij} \quad \text{for } i \neq j \tag{1a}$$

and

$$\epsilon_{ii} = \frac{1}{E}\,[\sigma_{ii}-\nu(\sigma_{jj}+\sigma_{kk})] \tag{1b}$$

In solving two-dimensional problems (i.e., plane-stress), it is generally assumed that the stress field complies with the Airy stress function (ref 3), ϕ, (Eq. (2a))

$$\frac{\partial^4\phi}{\partial x^4} + 2\,\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} = 0 \tag{2a}$$

where, in the absence of body forces,

$$\sigma_{xx} = \frac{\partial^2\phi}{\partial y^2} \;,\quad \sigma_{yy} = \frac{\partial^2\phi}{\partial x^2} \;,\quad \text{and} \quad \sigma_{xy} = -\,\frac{\partial^2\phi}{\partial x\cdot\partial y} \tag{2b}$$

In the problem at hand, due to the body's axisymmetry and due to axisymmetrical loading, it can be shown (ref 3) that the Lamé expression (Eqs. (3a) and (3b)) of the stress field in an elastic tube satisfies the Airy stress function.

$$\sigma_{\theta\theta} = -\,\frac{\left[\left(\frac{b}{a}\right)^2 + \left(\frac{b}{r}\right)^2\right] p_o - \left[\left(\frac{b}{r}\right)^2 + 1\right] p_i}{\left(\frac{b}{a}\right)^2 - 1} \tag{3a}$$

[2] A. E. H. Love, A Treatise on the Mathematical Theory of Elasticity, Fourth Edition, Dover Publication, New York, 1944, pp. 102-103.
[3] S. Timoshenko and J. N. Goodier, Theory of Elasticity, Second Edition, Engineering Societies Monographs, 1951.

and

$$\sigma_{rr} = - \frac{\left[\left(\frac{b}{a}\right)^2 - \left(\frac{b}{r}\right)^2 \right] p_O + \left[\left(\frac{b}{r}\right)^2 - 1 \right] p_i}{\left(\frac{b}{a}\right)^2 - 1} \tag{3b}$$

In the absence of external pressure, $p_O = 0$, Eqs. (3a) and (3b) are equivalent to the authors' (ref 1) Eqs. (2a) and (2b), respectively. Once plastic deformation commences however, other assumptions concerning the material's behavior have to be made. The most common ones are that when plastic deformation takes place, either Tresca's yield criterion or Mises' yield criterion prevails. Tresca's yield criterion is given in Eq. (4) which is equivalent to the authors' (ref 1) Eq. (3)

$$\left| \sigma_{\theta\theta} - \sigma_{rr} \right| = \sigma_Y \tag{4}$$

and Eq. (5) expresses Mises' yield criterion (equivalent to Eq. (9) in the subject paper (ref 1)).

$$\sqrt{\tfrac{1}{2}[(\sigma_{\theta\theta}-\sigma_{rr})^2 + (\sigma_{rr}-\sigma_{zz})^2 + (\sigma_{zz}-\sigma_{\theta\theta})^2]} = \sigma_Y \tag{5}$$

Tresca's yield criterion is based on the assumption that material flows plastically when a resolved shear stress, on a plane inclined at a $\pi/4$ angle to the two principal directions, reaches a critical value. Mises' yield criterion is derived from a stress function that is independent of the coordinate system's orientation (a stress invariant) and thus assumes the material's isotropy (ref 4). It also satisfies a condition whereby the total strain energy reaches a critical value. Thus, inherent to these two criteria, as they are expressed

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

[4]Betzalel Avitzur, Metal Forming: Processes and Analysis, McGraw-Hill Book Company, 1968, Chapter 2.

mathematically in this report, is the assumption that the material is nonstrain-hardening (or nonstrain-softening) and isotropic (which excludes the Bauschinger effect as well).

In the absence of such equations, as Hooke's law, for the plastically deforming material, while certain continuities in strain and stress have to be satisfied, exact solutions for such problems are, in general, difficult to obtain (ref 5). However, in problems such as beam bending and autofrettage where the plastic deformation is being constrained by the elastic portion of the subject body, some solutions can be offered. These solutions depend on the assumed yield criterion in the plastic region. A solution is presented for the Tresca yield criterion in Eqs. (4) through (8) of the subject paper (ref 1) (Eqs. (9), (8a), (8b), (6a), and (6b), respectively, of this discussion).

In a partially deformed (plastically) tube with the elastic-plastic interface at $r = c$, one can treat the outer sleeve of the tube, $c \leqslant r \leqslant b$, as an elastic tube, where the condition at $r = c$ complies with the selected yield criterion and yet satisfies Hooke's law. For a tube subjected to internal pressure only, if Tresca's yield criterion is considered at $r = c$, the Lamé solution will offer

$$\sigma_{\theta\theta} = \tfrac{1}{2}\left[\left(\frac{c}{b}\right)^2 + \left(\frac{c}{r}\right)^2\right]\sigma_Y \tag{6a}$$

and

$$\sigma_{rr} = \tfrac{1}{2}\left[\left(\frac{c}{b}\right)^2 - \left(\frac{c}{r}\right)^2\right]\sigma_Y \tag{6b}$$

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[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

[5]Betzalel Avitzur, Metal Forming: Processes and Analysis, McGraw-Hill Book Company, 1968, Chapters 4 and 5.

as the stress field in the elastic range, $c \leqslant r \leqslant b$. As shown by Manning (ref 6), complying with Eq. (7) (Eq. (1) in the subject paper (ref 1)) is a prerequisite for satisfying equilibrium throughout the tube's cross section.

$$\frac{d\sigma_{rr}}{\sigma_{\theta\theta} - \sigma_{rr}} = \frac{dr}{r} \tag{7}$$

In the elastic region, Lamé's solution satisfies this requirement automatically (ref 3). However, Eq. (7) has to be satisfied together with the selected yield criterion in the plastic region, $a \leqslant r \leqslant c$, as well. And, indeed, the authors' (ref 1) Eqs. (5) and (6) (rewritten here as Eqs. (8a) and (8b)) are the solutions to Eq. (7) of this report with $\sigma_{\theta\theta} - \sigma_{rr}$ = constant (Tresca's yield criterion), when one uses Eqs. (6a) and (6b) (Eqs. (7) and (8) (ref 1)) as the boundary condition at $r = c$.

$$\sigma_{\theta\theta} = [\ell n(\frac{r}{c}) + \tfrac{1}{2}(1 + (\frac{c}{b})^2)] \cdot \sigma_Y \tag{8a}$$

and

$$\sigma_{rr} = [\ell n(\frac{r}{c}) - \tfrac{1}{2}(1 - (\frac{c}{b})^2)] \cdot \sigma_Y \tag{8b}$$

Setting $r = a$ (and reversing the sign) will yield

$$p_i = -\sigma_{rr(a)} = [\ell n\ \frac{c}{a} + \tfrac{1}{2}(1-(\frac{c}{b})^2)] \cdot \sigma_Y \tag{9}$$

(where $\sigma_{rr(a)} \equiv \sigma_{rr}$ @ $r = a$) which is the internal pressure (at the bore) required to reach yielding (according to Tresca's criterion) at $r = c$. (Equation (9) of this report is the same as the authors' (ref 1) Eq. (4).)

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[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

[3]S. Timoshenko and J. N. Goodier, Theory of Elasticity, Second Edition, Engineering Societies Monographs, 1951.

[6]W. R. D. Manning, "The Overstrain of Tubes by Internal Pressure," Engineering, Vol. 159, 1945, pp. 101-102, 183-184.

In plane-strain, $\epsilon_{zz} = 0$, according to Hooke's law (Eq. (1))

$$\sigma_{zz} = \nu(\sigma_{rr}+\sigma_{\theta\theta}) \tag{10}$$

in the elastic range. It is logical to assume incompressibility in the plastic region, however, Eq. (10) has been derived for the elastic region and it should not be used where plastic deformation prevails, regardless of the assumed criterion (except at the elastic-plastic interface). At the elastic-plastic interface $r = c$, this equation should be used with the material's actual Poisson's factor, ν, which is usually between $\nu = 0.25$ and $\nu = 0.35$ (far from 0.5). Thus, the authors' (ref 1) use of their Eq. (11)

$$\sigma_{\theta\theta} - \sigma_{rr} = \frac{2}{\sqrt{3}} \sigma_Y$$

amounts to assigning a yield strength which is about 15.5 percent higher than the actual yield strength to the already solved solution when Tresca's yield criterion is assumed. It has no relevance to an assumed Mises' yield criterion, either in plane-stress or in plane-strain.

UPPER AND LOWER BOUND SOLUTIONS AT PEAK LOAD VERSUS RESIDUAL STRESSES

Lode (ref 7) has demonstrated that Mises' yield criterion in plane-stress deviates from Tresca's by no more than a factor of $2/\sqrt{3} \cong 1.155$. However, this does not imply that seeking an answer to the predicted residual stresses after autofrettage, assuming that Mises' criterion prevails, leads to an answer which is within 15.5 percent of that which is obtained by assuming that Tresca's yield criterion prevails. By assuming a yield strength which is larger by a factor of $2/\sqrt{3}$ to Tresca's solution (which the authors (ref 1) erroneously call Mises'

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

[7]W. Lode, "Versuche über den Einfluss der mittleren Hauptspannung auf das Fliessen der Metalle Eisen, Kupfer und Nickel," Z. Physik, Vol. 36, 1926, pp. 913-939.

solution in plane-strain), their residual hoop stresses at the bore differed by a factor of 2. If the computations are based on pressurization to the same elastic-plastic interface, $r = c$, then this can be explained by reasoning that the residual stress is the difference between two large numbers (stresses under pressure minus the elastic recovery). Thus, while these two numbers for stress under load might differ by 15.5 percent (relative to each other), this slight difference constitutes a larger deviation in proportion to their difference after subtracting the elastic recovery. If, however, one compares the two results when the pressurization load is the same--then the factor by which the resultant residual stresses differs is of no consequence. That is, if the pressure brings about a plastic region, $r = c > a$, no matter how small considering the lower yield strength, but no yielding considering the higher yield strength, then no matter how small the residual stress is in the first case, it is larger by a factor of infinity relative to no residual stresses at all. Based on the above, one might argue that since Tresca'a yield criterion establishes a lower bound and Mises' yield criterion cannot exceed it by more than 15.5 percent, the two Tresca solutions, offered by the authors (ref 1), constitute a lower and an upper bound solution. However, this is not necessarily the case either.

Setting $r = c$ in Eqs. (6a) and (6b) above (Eqs. (7) and (8) in the subject paper (ref 1)) will yield

$$\sigma_{\theta\theta}(c) = \tfrac{1}{2}\left[\left(\frac{c}{b}\right)^2 + 1\right] \cdot \sigma_Y = \frac{\left(\frac{b}{c}\right)^2 + 1}{2\left(\frac{b}{c}\right)^2} \cdot \sigma_Y \tag{11a}$$

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

and

$$\sigma_{rr(c)} = \tfrac{1}{2}[(\frac{c}{b})^2 - 1] \cdot \sigma_Y = -\frac{(\frac{b}{c})^2 - 1}{2(\frac{b}{c})^2} \cdot \sigma_Y \tag{11b}$$

as the state of stress at the elastic-plastic interface, $r = c$, assuming Tresca's yield criterion. Integrating Eq. (7) between $r = c$ and $r = a$, will yield the internal pressure at the bore, $r = a$, assuming that Tresca's yield criterion prevails in the plastic region (to be consistent with the assumption that led to Eqs. (6a) and (6b) and consequently to Eqs. (11a) and (11b)), and using Eqs. (6a) and (6b) for $\sigma_{\theta\theta}$ and σ_{rr}. This pressure has been computed in Eq. (9) above (the authors' (ref 1) Eq. (4)). However, if one assumes that yielding follows Mises' criterion, then it can be shown (ref 8) that in plane-stress and in the absence of external pressure, $p_o = 0$, the stresses at the elastic-plastic interface are

$$\sigma_{\theta\theta(c)} = \frac{(\frac{b}{c})^2 + 1}{\sqrt{3(\frac{b}{c})^4 + 1}} \cdot \sigma_Y \tag{12a}$$

and

$$\sigma_{rr(c)} = -\frac{(\frac{b}{c})^2 - 1}{\sqrt{3(\frac{b}{c})^4 + 1}} \cdot \sigma_Y \tag{12b}$$

As $b/c \to 1$, these values approach those obtained for Tresca's yield criterion (Eq. (11)), and as $b/c \to \infty$, the ratio between the two solutions approaches $2/\sqrt{3}$

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.
[8]R. Weigle, "Elastic-Plastic Analysis of a Cylindrical Tube," Technical Report WVT-RR-6007, Watervliet Arsenal, Watervliet, NY, March 1960.

in compliance with Lode (ref 7).

$$\frac{|\sigma_{rr}| \text{ @ yield for Mises' yield criterion in plane-stress}}{|\sigma_{rr}| \text{ @ yield for Tresca's yield criterion}} = \frac{2}{\sqrt{3 + (\frac{c}{b})^4}} \tag{13}$$

where

$$1 \leq \frac{2}{\sqrt{3 + (\frac{c}{b})^4}} \leq \frac{2}{\sqrt{3}}$$

However, solving the differential Eq. (7) with the assumed Tresca yield criterion gives

$$\ell n(\frac{a}{c}) = \frac{(\frac{b}{c})^2 - 1}{2(\frac{b}{c})^2} - \frac{p_i}{\sigma_Y} \tag{9'}$$

for the corresponding internal pressure, p_i. Whereas, assuming Mises' criterion in plane-stress (ref 8) and Eq. (12) as the boundary condition at $r = c$, gives

$$\ell n \frac{a}{c} = \frac{1}{4} \left\{ \ell n \frac{\left[\sqrt{3} \cdot \sqrt{\frac{4}{3} (\frac{\sigma_Y}{p_i})^2 - 1} + 1\right]^2}{4(\frac{\sigma_Y}{p_i})^2} - \ell n \frac{4(\frac{b}{c})^4}{3(\frac{b}{c})^4 + 1} \right.$$

$$\left. - 2\sqrt{3} \left[\tan^{-1}\sqrt{\frac{4}{3}(\frac{\sigma_Y}{p_i})^2 - 1} - \tan^{-1} \frac{3(\frac{b}{c})^2 + 1}{\sqrt{3}\left[(\frac{b}{c})^2 - 1\right]} \right] \right\} \tag{14}$$

[7] W. Lode, "Versuche über den Einfluss der mittleren Hauptspannung auf das Fliessen der Metalle Eisen, Kupfer und Nickel," Z. Physik, Vol. 36, 1926, pp. 913-939.

[8] R. Weigle, "Elastic-Plastic Analysis of a Cylindrical Tube," Technical Report WVT-RR-6007, Watervliet Arsenal, Watervliet, NY, March 1960.

for the internal pressure, p_i, at $r = a$. Thus, not only is the stress at $r = c$ different, but so is the pressure at the bore that causes it, and not by the same proportion. Thus, the elastic recovery computed when Tresca's yield criterion is assumed differs from that which is being computed when Mises' yield criterion in plane-stress is assumed.

MISES' SOLUTION IN PLANE-STRAIN

Lamé's equations satisfy Airy's equation and thus they apply to plane-stress problems. Yet pressure vessels are very long relative to their diameter, and therefore do not fit the category of plane-stress conditions. However, the Lamé equations yield a uniform strain in the axial direction

$$\epsilon_{zz} = \frac{\nu}{E}\,(\sigma_{rr}+\sigma_{\theta\theta}) = -\frac{2\nu}{E}\,\frac{\left(\frac{b}{a}\right)^2 \cdot p_o + p_i}{\left(\frac{b}{a}\right)^2 - 1} \tag{15}$$

which is independent of the coordinates. Therefore, if a physical constraint of $\epsilon_{zz} = 0$ is imposed, the stress distribution throughout the elastic region will be uniform. It can be shown (ref 9) that if the Lamé equations are assumed to hold in plane-strain, yielding will take place at $r = c$ when

$$\sigma_{\theta\theta(c)} = \frac{\left(\frac{b}{c}\right)^2 + 1}{\sqrt{3\left(\frac{b}{c}\right)^4 + (1-2\nu)^2}} \cdot \sigma_Y \tag{16a}$$

and

$$\sigma_{rr(c)} = -\frac{\left(\frac{b}{c}\right)^2 - 1}{\sqrt{3\left(\frac{b}{c}\right)^4 + (1-2\nu)^2}} \cdot \sigma_Y \tag{16b}$$

[9]Boaz Avitzur, unpublished.

with

$$\sigma_{zz(c)} = \frac{2\nu}{\sqrt{3\left(\frac{b}{c}\right)^4 + (1-2\nu)^2}} \cdot \sigma_Y \tag{16c}$$

This reviewer applied the above equations to Eq. (7) as boundary conditions and assumed that due to the elastic strain in the plastic region while under load, the same ratio between σ_{zz} and $\sigma_{\theta\theta} + \sigma_{rr}$ also prevailed throughout the plastic region. He followed Weigle's (ref 8) procedure and arrived at Eq. (17) correlating the pressure, p_i, at the bore with an elastic-plastic interface at $r = c$

$$\ell n \frac{a}{c} = \frac{1}{4} \cdot \left\{ \ell n \frac{\left[\sqrt{\frac{3}{\eta}} \cdot \sqrt{\frac{4\delta}{3\eta}\left(\frac{\sigma_Y}{p_i}\right)^2 - 1} + 1\right]^2}{4 \cdot \frac{\delta}{\eta^2} \cdot \left(\frac{\sigma_Y}{p_i}\right)^2} - \ell n \frac{4 \cdot \left(\frac{b}{c}\right)^4}{3\left(\frac{b}{c}\right)^4 + \eta} \right.$$

$$\left. + 2\sqrt{\frac{3}{\eta}} \left[\tan^{-1} \sqrt{\frac{4\delta}{3\eta}\left(\frac{\sigma_Y}{p_i}\right)^2 - 1} - \tan^{-1} \frac{3\left(\frac{b}{c}\right)^2 + \eta}{\sqrt{3\eta}\left[\left(\frac{b}{c}\right)^2 - 1\right]}\right] \right\} \tag{17}$$

where $\delta = 1 - \nu + \nu^2$ and $\eta = (1-2\nu)^2$. Clearly, the internal pressure required for yielding at $r = c$ under plane-strain conditions (with the above-mentioned assumptions) differs somehow from that required under plane-stress conditions (Eq. (14)) and differs significantly from that computed for the Tresca yield criterion (Eq. (9')).

[8]R. Weigle, "Elastic-Plastic Analysis of a Cylindrical Tube," Technical Report WVT-RR-6007, Watervliet Arsenal, Watervliet, NY, March 1960.

YIELDING ON PRESSURIZATION VERSUS REVERSE YIELDING

In each set of assumptions considered above, Tresca's yield criterion or Mises' yield criterion in plane-stress or in plane-strain, if and when yielding is due to an internal pressure, the tangential (or hoop) component of the stress and its radial component are of the opposite signs (the tangential component is tensile, while the radial component is compressive). However, upon depressurization as well as under external pressure, yielding commences when both the tangential and the radial components of the stress are compressive. Therefore, the equations used for the determination of the stress at the elastic-plastic interface, $r = c$, upon pressurization (Eq. (11) for Tresca's yield criterion, Eq. (12) for Mises' yield criterion in plane-stress, and Eq. (16) for Mises' yield criterion in plane-strain), do not apply for reverse yielding (with or without the incorporation of the Bauschinger effect). Hence, the derivation of Eqs. (13) through (17) from Eq. (5) (in the subject paper (ref 1)) for reverse yielding, assuming Tresca's yield criterion, is inappropriate. Furthermore, $\sigma_{\theta\theta} - \sigma_{rr}$ in Eq. (7) ceases being a constant if Tresca's yield criterion is considered.

Moreover, the authors imply that the (compressive) yielding upon unloading has to be lower than yielding in tension (Bauschinger effect) for reverse yielding to take place. Yet it can be shown (ref 9) that when Mises' yield criterion is assumed, and depending on the tube's wall thickness-to-bore ratio and on its fraction that undergoes plastic deformation upon pressurization, even in an elastic-perfectly plastic and isotropic material, reverse plastic deformation

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.
[9]Boaz Avitzur, unpublished.

can be anticipated. Furthermore, due to the presence of residual compressive radial stresses at the interior of the tube's wall and due to their vanishing towards the bore, higher residual tangential (hoop) stresses are attained beneath the bore's surface than at the bore itself (even without accounting for a possible Bauschinger effect).

STRAIN-HARDENING AND BAUSCHINGER EFFECT ON YIELDING

The authors (ref 1) imply that they employed Manning's method (ref 6) for the inclusion of strain-hardening. Flaws in Manning's method* not withstanding, this author is questioning the need for using an elaborate method, such as Manning's, designed for strains in excess of 20 percent and far beyond full plasticity at the tube's outer diameter, to solve a problem of partially plastic-partially elastic autofrettage. Moreover, since Manning's method employs an actual stress-strain curve for strain-hardening, how is it used when the authors (ref 1) replace the material's yield strength, σ_Y, by $0.5*(\sigma_U+\sigma_Y)$, where σ_U = material's ultimate strength?

Furthermore, strain-hardening and the Bauschinger effect are, as the former term implies, functions of the amount of prior plastic deformation. As such, neither of them applies to the elastic-plastic interface, but rather to the

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*Manning's elaborate computations are based on an assumption that the radial stress, $\sigma_{rr(\rho)}$, at $r = \rho$ (where $r_i = a \leq \rho \leq r_o = b$) is the negative of the difference between the pressure p_i at the bore of a tube with wall ratio of (b/a) and p_ρ at the bore of a tube with the wall ratio of (b/ρ) for the same displacement u_ρ or for the same strain u_ρ/ρ at $r = \rho$. This assumption is neither in agreement with Eq. (7) (Manning's (ref 6) Eq. (8) and the authors' (ref 1) Eq. (1))--the validity of which Manning has so eloquently proven--nor does it concur with Lamé's full solution (one that includes external pressures) for the elastic range.

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

[6]W. R. D. Manning, "The Overstrain of Tubes by Internal Pressure," Engineering, Vol. 159, 1945, pp. 101-102, 183-184.

plastic region, $a \leqslant r \leqslant c$, and they increase progressively from zero at $r = c$ to a maximum at $r = a$. These increases (or decreases in the case of the Bauschinger effect) depend on the amount of plastic strain at $r = a$ and in the interval $a \leqslant r \leqslant c$. Modifying the yield strength in the respective Eqs. (11), (12), or (16) amounts to artificially changing the boundary condition in the solution to Eq. (7) instead of applying the appropriate function for $\sigma_{\theta\theta} - \sigma_{rr}$ in the pertaining range, $a \leqslant r \leqslant c$.

PREDEFORMATION VERSUS POST-AUTOFRETTAGE RESIDUAL STRESSES

The authors (ref 1) were careful enough to check the tube for pre-existing stresses in order to delete their effect on the post-autofrettage residual stresses. However, this author is questioning the way they did it. In order to account for the effect of the pre-existing stresses on the post-autofrettage residual stresses, they simply subtract the former from those measured after autofrettage--suggesting that this difference represents the actual residual stresses due to autofrettage Unfortunately, however, pre-existing stresses affect the commencement of yielding at the elastic-plastic interface as this surface sweeps through the wall's thickness. They also affect the elastic state of stress at the peak of pressurization, which is responsible for the post-pressurization elastic recovery. They do not, however, have any effect within the plastic region, once the elastic-plastic front sweeps through. Moreover, pre-existing stresses affect yielding when added to those imposed by pressurization (internally and/or externally), and since yielding is a function of two or three of the principal stress components (depending on the yield criterion adapted), one should consider all six components of the pre-existing stresses.

[1]A. Stacey and G. A. Webster, "Determination of Residual Stress Distribution in Autofrettaged Tubing," International Journal of Pressure Vessels and Piping, Vol. 31, 1988, pp. 205-220.

Likewise, the contribution of such stresses in the elastic range should be added to those responding to the autofrettaging pressure for the computation of the elastic recovery, rather than being subtracted from those observed as post-autofrettage residual stresses. This can be illustrated best by analyzing the residual stresses in a tube that has been autofrettaged 100 percent and beyond. In this case, pre-autofrettage stresses will have no effect on the post-autofrettage residual stresses.

SUMMARY

While making assumptions and relying on approximations are inevitable in most engineering problems, one should try to determine the range of the possible errors that might result from such reliance. An attempt was made here to identify some of the assumptions commonly used in the analyses of autofrettage residual stresses and to estimate their effect on the accuracy of the results. Two alternative methods of computation--using Mises' yield criterion--were offered here, without their derivations.

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